QP CODE: 24044653

24044653

Reg No : ....

Name : ....

M.Sc DEGREE (CSS) EXAMINATION, OCTOBER 2024

Third Semester

M.Sc MATHEMATICS , M.Sc MATHEMATICS (SF)

CORE - ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

2019 ADMISSION ONWARDS

6BDA4900

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Find the integral curves of* $\frac{dx}{x^2(y^3-z^3)} = \frac{dy}{y^2(z^3-x^3)} = \frac{dz}{z^2(x^3-y^3)}$

2. *Verify that the equation* $yz(y+z)\,dx + xz(x+z)\,dy + xy(x+y)\,dz = 0$ *is integrable.*

3. What is the general form of the linear partial differential equation in n variables. Explain how a general solution of this equation is found.

4. Verify that the equation $z = \sqrt{(2x+a)} + \sqrt{2y+b}$ is a complete integral of the partial differential equation $z = \frac{1}{p} + \frac{1}{q}$.

5. What is meant by compatible systems of first order equations? State the condition under which the partial differential equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ are compatible?

6. Find the complementary function of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

7. Prove $F(D, D')e^{ax+by} = F(a, b)e^{ax+by}$.

8. Find the particular integral of $[D^2 - D']z = 2y - x^2$.

9. Establish a formula for finding the potential function of a family of equipotential surfaces.

10. Show that the real and imaginary parts of an analytic function are harmonic

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Find the equations of the system of curves on the cylinder* $2y = x^2$ *orthogonal to its intersections with the hyperboloids of the one-parameter system* $xy = z + c$

Turn Over

12. Eliminate the arbitrary function f from the given equations.
   a) $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$
   b) $z = xy + f(x^2 + y^2)$

13. Find a complete integral of the equation $p^2x + q^2y = z$.

14. Show that the differential equation $2xz + q^2 = x(xp + yq)$ has a complete integral $z + a^2x = axy + bx^2$ and deduce that $x(y + hx)^2 = 4(z - kx^2)$ is also a complete integral.

15. By Jacobi's method, solve $z^2 = pqxy$.

16. Verify that the PDE $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x^2}$ is satisfied by $z = \frac{1}{x}\phi(y - x) + \phi'(y - x)$.

17. By separating the variables show that the equation $\nabla_1^2 V = 0$ has solutions of the form $Aexp(nx \pm iny)$ where A and n are constants.Deduce that the functions of the form $V(x,y) = \sum_r A_r e^{\frac{-r\pi x}{a}} sin\frac{r\pi y}{a}$, $x \geq 0, 0 \leq y \leq a$ where $A_r's$ and $B_r's$'s are constants, are the plane harmonic functions satisfying the conditions $V(x,y) = 0, V(x,a) = 0, V(x,y) \to 0$ as $x \to \infty$

18. Show that in cylindrical coordinates ρ, z, ϕ , the Laplace's equation has solutions of the form $R(\rho)exp(\pm mz \pm in\phi)$ where $R(\rho)$ is a solution of Bessel's equation $\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} + (m^2 - \frac{n^2}{\rho^2})R = 0$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Prove the following.
   a) A Pfaffian differential equation in two variables always possesses an integrating factor.
   b) A necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$ a relation $F(u,v) = 0$ not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$.

20. a) Find the general solution of the equation $2x(y + z^2)p + y(2y + z^2)q = z^3$ and deduce that $yz(z^2 + yz + -2y) = x^2$ is a solution.
   b) Find the general integral of the equation $(x - y)p + (y - x - z)q = z$ and the particular solution through the circle $z = 1, x^2 + y^2 = 1$.

21. Reduce the equation to canonical form and solve $u_{xx} + 2u_{xy} + u_{yy} = 0$.

22. (a) State and prove the necessary condition that a family of surfaces $f(x,y,z) = c$ is a family of equipotential surfaces.
   (b) Show that the surfaces $(x^2 + y^2)^2 - 2a^2(x^2 - y^2) + a^4 = c$ can form a family of equipotential surfaces and find the general form of the corresponding potential function.

(2×5=10 weightage)